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The Mystery of Pythagoras' Comma

One of the greatest of the mysteries of the science of sound is the strange phenomenon known as Pythagoras' comma, which since time immemorial has been to man a symbol of his fallen state of imperfection.

Given that different pitches of tone arrange themselves into octaves (which have the ratio of 2:1), man from the earliest times needed to find a method whereby the notes within each octave could be calculated. This could be done, it was discovered, by using the next most fundamental and harmonious tonal ratio, that of 3:2, or the interval known in music as a pure fifth. If one pictures the octave as a circle, then the interval of a fifth represents a seven-twelfths circuit of the circle, from 12 o'clock, moving clockwise, to 7 o'clock. This gives one pitch. By continuing around clockwise again, another pitch is produced, but not at 2 o'clock on the same circle, for the 'circle' is actually not a circle at all, but a section of a spiral which spirals upward into higher octaves. By going around clockwise in a series of perfect fifths, a series of twelve notes can be produced, related by the vibratory ratios of the series 1, 3:2, (3:2), and so on to (3:2). Try this out for yourself: by advancing twelve times around a clock face in jumps of seven hours at a time, you will end up back at 12 o'clock, having landed once on each hour.

However, these twelve notes could not practically be used as the scale of a system of music, since they extend over several octaves with wide gaps in between each note. Therefore the ancient philosopher-musicians devised schemes whereby intervals of a fifth could be used to calculate twelve notes within one octave. For instance, the Chinese went up in pitch by the ratio of 1:3, then down by 3:1, up by 1:3, and so on.

However, given these twelve notes, if a thirteenth note was attempted, in order to complete the octave with a series of twelve intervals of a fifth, it was found that the octave could not be perfectly completed! This is a universal law of the physics of tone and, indeed, of mathematics: a cycle of twelve fifths completes seven octaves (note the mystical numbers: twelve and seven) plus a little more. Expressed mathematically: $(3:2)^{12} > (2:1)^7$ by a slight excess interval. This interval, since Pythagoras was one who noted it, has become known as Pythagoras' comma. The difference is a ratio of about 80:81, the extra 1 being the comma.

The comma produces huge cosmological, as well as practical, implications and results.

Since this system of calculating twelve notes does not perfectly complete the octave, the specific twelve pitches it produces are imperfect for use together in harmony. They do not perfectly harmonise since they do not divide the octave into perfectly accurate divisions of twelve as, say; the hours of a clock do perfectly divide a clock face.

Yet to adjust their slight pitch discrepancies in order to produce perfect harmony would render each note imperfect as an interval of a fifth, or as a ratio of 3:2. The perfecting of their musical system was paramount to the ancients, for their music had to harmonise with the eternal laws of the universe. Therefore the perfect ratio of 3:2 and its interval of a perfect fifth were regarded as sacred and inviolable. If the system resulted, as it did, in the harmony of mortal music being imperfect, then this had to be borne as a manifest symbol of man's fall from grace, and of the inherent imperfection of the non-heavenly realm of time and space. Indeed, perhaps the ancients were correct in this, for the comma is a strange and wide-ranging phenomenon, being literally 'written into' the physical and mathematical laws of the universe. We are entering here into regions of thought difficult for the Western mind to grasp, but the possibility seems to be that it is by the phenomenon of Pythagoras' comma that the very nature of our lives within the realm of mortality is arranged.

Though the comma may be a symbol of the imperfection of man's mortal state, the very same comma simultaneously provides the way back to the original state of perfection. For the comma is not a slight interval less than seven octaves, but in excess of them. In the ancient world this fact was widely conceived as a symbol of renewal. The cycle of twelve perfect fifths did not close and finish a cycle of seven octaves, but exceeded it, and thus, as it were, spiralled upward. There is evidence that this upward spiral of renewal was mystically associated with the widespread ancient myth of the phoenix, the archetypal 'bird' which is resurrected from its own ashes. Pythagoras' comma, then, can be seen as being God's own anagram written into the very laws of the universe and of physics. It is by the nature of this anagram that man is heir to the promise of eventual resurrection and ascension out of the dim caverns of mortality.

Through the ages civilizations have often wrestled with the dilemma of whether to put up with imperfect musical pitches while retaining pure idealism. In their system of pitch-calculation, or else to depart a little from alignment with the heavenly ideal by taking the practical path of slightly adjusting their notes in order to harmonize them. Though there were individual figures who dissented, the ancient cultures generally opted, in their idealism, for heavenly alignment at the expense of having slightly imperfect relationships between their notes. However, by the seventeenth century AD the Western world had begun its entry into the present era of science and logic. This era was marked not only by an attitude of over-materialism, but also by a most useful pragmatism. This pragmatism entered into the debate on tuning, and the controversy heightened: should man opt for heavenly perfection and musical inharmonic, or for a departure from abstract idealism for the sake of expanding music's harmonic possibilities. (Also, on the very down-to-earth level, the construction of instruments such as keyboards according to the precepts of idealism was proving insuperably problematic in some aspects: a great many more keys were necessary in order to incorporate the many 'extra' notes created by a cycle of perfect fifths.)

The idea of man's imperfect state being related to a tonal mis-alignment with the Above was widespread in the poetry of the seventeenth century. Clement Paman wrote:

*Screw thee high My heart: up to The Angels' key.
What if thy strings all crack and flye?
On such a Ground, Musick 'twill be to dy.*

George Herbert actually went so far as to associate man's tonal imperfection with the Crucifixion:

The Cross taught all wood to resound his Name, Who bore the same.
His stretched Sinews taught all strings, what key
Is best to celebrate this most high Day.
Consort both heart and lute, and twist a song
Pleasant and long :
Or, since all music is but three parts vied And multiplied,
O let Thy blessed Spirit bear a part
And make up our defects with His sweet Art.

J. S. Bach became the first major musician to depart from the idealism of tradition, opting for the pragmatic approach of equal temperament, or octaves divided into twelve 'equal' divisions. This opened up vast new avenues of harmonic possibility, and led the way into the towering musical developments of the eighteenth and nineteenth centuries. The intensely religious Bach was as good as saying, 'Well, since we do find ourselves in this fallen state of mortal imperfection, we had better be practical about it, harmonize our music, and by this very process begin composing a new and better art form. An art of enhanced sublimity can in itself lead us back to the heavens.

This, in essence, is a summary of man's relationship and dealings with the phenomenon of Pythagoras, comma. But it is not the end of the entire story, for the heavenly bodies themselves have a final tale to add.

The months of the year were associated in antiquity with the total twelve notes of the musical octave. But there are, of course, two slightly different kinds of months: the solar or calendar month which averages 30.44 days, making a total of 365.256 days per year, and the synodic lunar month of 29.5306 days. The solar month is exactly one twelfth of the time it takes for the earth to orbit once around the sun. The synodic lunar month is the time that the moon takes to pass through all of its phases - say, from new moon to new moon. Though the lunar month is not exactly the same as a solar month, it comes remarkably close, with the curious result that there are, approximately twelve lunar months (yes, that number twelve again.) per calendar year.

Indeed, since solar months offer less obvious signs of their coming and going than do those of the moon, it has frequently been by the phases of the moon that cultures have measured the passing of time.

The reader will probably not have missed the noteworthy fact that this phenomenon of two different divisions of the year into two slightly different lengths of months strikes a very close parallel with Pythagoras' comma, with its two slightly different sets of twelve notes around the circle of an octave. That the parallel may be more than academic is

suggested by the fact that according to the ancient wisdom the year does represent the passage of the earth through an octave of twelve notes, or Cosmic Tones. Taking the parallel to its logical conclusion. The solar year, which totals the exact and accurate length of 365.256 days, equates with a cosmic form of equal temperament. By measuring time by the solar year rather than by the lunar year, man not only displays an obvious practicality, but also opens for himself the possibility of renewal and self evolution.

The two ratios - between the two years (solar and lunar) and the two types of musical scale (of perfect fifths and of equal tempera-ment) - are astonishingly close to each other. The ratio of Pythagoras' comma is 531441 to 524288, the former figure being larger than the latter by the factor of 1.01364. The ratio between the two years is 365.256 days to 354.3672, the former being the larger by a factor of 1.03073. To express the closeness of the two ratios in a more graphic way, we can translate the ratio of the comma into calendar terms. That is, 531441 is to the solar year of

365.256 days as 524288 is to the time period of 360.340 days - which is only six days out from the precise lunar year of 354 1/3 days.

As might be expected, the cultures of antiquity seem not to have missed this phenomenon of a 'cosmic comma'. Traditionally, the measurement of the year by means of solar months was considered holy and righteous; whereas to keep time by the moon was both incorrect and evil. This was one of the chief differences of opinion between the Pharisees and the Sadducees. Whereas the Sadducees marked time by the sun, the Pharisees marked time by the lunar year. Interestingly, however, this lunar year was not stated as being 364 1/3 days long, which it is, and which the Pharisees must easily have been able to observe that it was. Rather, it was dogmatically stated to be 360 days in length. This figure was a direct but coded reference to the cosmic Pythagoras' comma since, as we have calculated above, 360 is the nearest whole number of days to what an absolutely perfect incidence of the comma would turn out to be, at 360.340 days.

The cosmological and moral significance of the two different years is an important theme in the famous apocalyptic text, The book of Enoch. Certain sections of Enoch and especially chapters LXXII to LXXXII, sometimes called The Book of the Courses of Heavenly Luminaries, take this as their central subject. In fact, though establishment scholars have called this particular section 'uninteresting in the extreme, they have missed the point that it is actually, along with the following sections, a mystical text on the subject of astrology (which is taught to Enoch, according to the book, the heavenly being Uriel). In Enoch LXXXII we are informed that reason for it being evil and dangerous to reckon time by the n is that due to the moon's inaccuracy the entire civilization d become out of alignment with the cycles and seasons of the heavens. Verses 4 and 5 read:

Blessed are the righteous, blessed are all those who walk in the way of righteousness and sin not as the sinners in the reckoning of all their days in which the sun traverses the heaven ... Owing to them men shall be at fault and not reckon them in the whole reckoning of the year: yea, men shall be at fault, and not recognize them accurately.

Running right through the astrological sections of Enoch one finds the widespread

concept in antiquity that the sun embodied righteousness and perfection, whereas the moon represented mortality and error. The author of the work sought to ensure that his readers would align themselves to the cycles of the sun. But this alignment had both an outer, astronomical, and an inner, spiritual character. The exhortation for man to follow the solar year was imbued with deeper implications of heavenly attunement and spiritual renewal. And the very name, Enoch, became associated with these implications, as is evident from Genesis 5: 2 3-4, in which we are allegorically told: `And all the days of Enoch were three hundred and sixty and five years (i.e. the number of whole days in the solar year]: and Enoch walked with God: and he was not; for God took him'. That is, by attuning himself spiritually to the cycles and vibrations of the sun, and to the God behind the manifested sun, Enoch is said to have ascended into immortality. The number 365 is therefore an esoteric reference to the fact that by `walking with (being close in consciousness to) God', any man can `be not' (being no longer found in time and space), for God will take him (in the process and ritual of the ascension).

Pythagoras, comma is an eternal reminder to us of the ancient mystical concept that there exist two fundamental states or frequencies of being: the dense and physical state of mortality, and the state of immortality as attained by Enoch, Jesus Christ, and other great masters of East and West. And that it is the goal and destiny of all men to so purify and perfect themselves that they can be translated from the lower state to the higher. If the Word was made flesh in one man, the Word can be made flesh in all.

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